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Differential Equations - A realistic approach

Dr.A.Nirmala Jyothsna
Head, Dept of Physics & Electronics
Ch.S.D.St.Theresa's College For Women , Eluru
nirmala.akurati@gmail.com

Abstract: The paper reports on ongoing developmental research efforts to adapt the instructional design perspective of differential equations for realistic approach to the learning and teaching of collegiate students.It focuses on the applications of Differential equations in real life. Differential equations have a remarkable ability to predict the world around us. They are used in a wide variety of disciplines, from biology, economics, physics, chemistry and engineering. They can describe exponential growth and decay, the population growth of species or the change in investment return over time.There are well-known equations drawnf rom different scientific and technical disciplines. A sense of their importance maybe realized from their ability to mathematically describe, or model, real-life situations.The equations come from the diverse disciplines of demography, ecology, chemical kinetics, architecture, physics, mechanical engineering, quantum mechanics, electrical engineering, civil engineering, meteorology, and a relatively new science called *chaos*.The same differential equation may be important to several disciplines, although for different reasons. For example, demographers, ecologists, and mathematical biologists would immediately recognize their importance of *Malthusian law of population growth* . It is used to predict populations of certain kinds of organisms reproducing under ideal conditions. This paper focuses on application of differential equations in Population Growth and Decay Radio-Active Decay and Carbon Dating ,Series Circuits ,Survivability with AIDS, Draining a tank ,Economics and Finance ,Mathematics Police Women, Drug Distribution in Human Body Harvesting of Renewable Natural Resources ,Forensic Science etc to broaden the knowledge & understanding .

Keywords:differentialequations,mathematicalmodelling,populationgrowth,carbondating,applications,predatorprey,forensicscience,biology

I. INTRODUCTION

Differential Equations are the language in which the laws of nature are expressed. Understanding properties of solutions of differential equations is fundamental to much of contemporary science and engineering.

Differential equations have a remarkable ability to predict the world around us. They are used in a wide variety of disciplines, from biology, economics, physics, chemistry and engineering. They can describe exponential growth and decay, the population growth of species or the change in investment return over time.

The ordinary differential equations as used in mathematical modeling, to analyze and understand a variety of real-world problems. Among the civic problems explored are specific instances of population growth and over-population, over-use of natural resources leading to extinction of animal populations and the depletion of natural resources, genocide, and the spread of diseases, all taken from current events. While mathematical models are not perfect predictors of what will happen in the real world, they can offer important insights and information about the nature and scope of a problem, and can inform solutions.

DIFFERENTIAL EQUATIONS IN PHYSICS

Mechanics:

$V = dr/dt$

$a = dv/dt = d^2r/dt^2$

$F = m\, dv/dt = m\, d^2r/dt^2$

Equations of motion

$V = U + at$

$S = ut + 1/2\, at^2$

$V^2 - u^2 = 2as$

1. **Newton's Low of Cooling**

$$\frac{dT}{dt} = \kappa\left(T - T_s\right)$$

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2. Growth and Decay

$$\frac{dy}{dt} = \kappa\, y$$

y is the quantity present at any time

3.Simple harmonic motion

We look at Simple Harmonic Motion in Physics

$$\frac{d^2x}{dt^2} = -\frac{k}{m}\,x$$

where x is the displacement from equilibrium of the mass m at time t, and k is the stiffness of the spring to which the mass is attached.

$d^2x/dt^2 + \omega^2 x = 0$, where $\omega^2 = k/m$.

Equation for damped oscillation is

$d^2x/dt^2 + \beta\, dx/dt + \omega^2 x = 0$, where $\omega^2 = k/m$ and where $\beta > 0$.

Equation for forced oscillation

$d^2x/dt^2 + \beta\, dx/dt + \omega^2 x = F_0 \sin\omega t$.

4.Thermodynamics:

In thermodynamics we use differential equations to find out

Thermal capacity - dQ/dt
Heat capacity (isobaric) - dH/dt
Specific heat at constant volume - $C_V = (dQ/dt)_V$
Specific heat at constant Pressure - $C_P = (dQ/dt)_P$

we also find Gibb's Function , Helm holtz function , Enthalpy using differential equations .

Maxwell's Thermodynamical relations are established using partial differentiation

Newton's law of cooling is an important application of D.E

5.Electricity & Electromagnetism :

Faraday's laws of Electromagnetic induction involve differential equation

$$e = -N\, d\varphi/dt$$

6.LR Series circuit

By Kirchhoff's second law the sum of the voltage drop across conductor and the voltage drop across the resistor (iR) is the same as the impressed voltage (E(t)) on the circuit. Current at time t, i(t), is the solution of the differential equation

$$I\frac{di}{dt} + Ri = E(t)$$

Applications of Differential Equations in Real life

Differential equations have a remarkable ability to predict the world around us. They are used in a wide variety of disciplines from

- Biology,
- Economics,
- Physics,
- Chemistry and Engineering.

They can describe exponential growth and decay, the population growth of species or the change in investment return over time.

Population Growth and Decay

The differential equation for rate of change of population is

$$\frac{dN(t)}{dt} = kN(t)$$

where N(t) denotes population at time t and k is a constant of proportionality, serves as a model for population growth and decay of insects, animals and human population at certain places and duration

Solution of this equation is $N(t)=Ce^{kt}$, where C is the constant of integration:

Integrating both sides we get $\ln N(t)=kt+\ln C$ or $N(t)=Ce^{kt}$

C can be determined if N(t) is given at certain time.

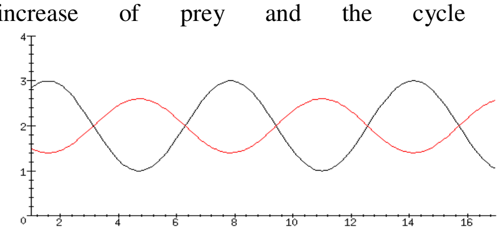
Predator-Prey Relationships

By: Maria Casillas, Devin Morris, John Paul Phillips, Elly Sarabi, & Nernie Tam

Formulation of the Scientific Problem

- There are many instances in nature where one species of animal feeds on another species of animal, which in turn feeds on other things. The first species is called the predator and the second is called the prey.
- Theoretically, the predator can destroy all the prey so that the latter become extinct. However, if this happens the predator will also become extinct since, as we assume, it depends on the prey for its existence.

What actually happens in nature is that a cycle develops where at some time the prey may be abundant and the predators few. Because of the abundance of prey, the predator population grows and reduces the population of prey. This results in a reduction of predators and consequent increase of prey and the cycle continues.



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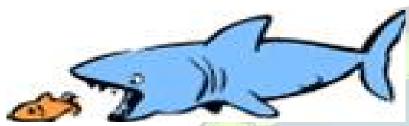
An important problem of *ecology* , the science which studies the interrelationships of organisms and their environment, is to investigate the question of coexistence of the two species.
To this end, it is natural to seek a mathematical formulation of this predator-prey problem and to use it to forecast the behavior of populations of various species at different times.

Risk and Food Availability

- Sharks appear to be a major threat to fish
- Availability of prey helps animals decide where to live



- **Predator-Prey Model: Fish & Sharks**
- We will create a mathematical model which describes the relationship between predator and prey in the ocean. Where the predators are sharks and the prey are fish.

In order for this model to work we must first make a few assumptions.
Assumptions
1. Fish only die by being eaten by Sharks, and of natural causes.
2. Sharks only die from natural causes.
3. The interaction between Sharks and Fish can be described by a function.

Differential Equations and how it relates to Predator-Prey
One of the most interesting applications of systems of differential equations is the predator-prey problem.
In this project we will consider an environment containing two related populations-a prey population, such as fish, and a predator population, such as sharks. Clearly, it is reasonable to expect that the two populations react in such a way as to influence each other's size.

- The differential equations are very much helpful in many areas of science.
- But most of interesting real life problems involve more than one unknown function. Therefore, the use of system of differential equations is very useful.

Without loss of generality, we will concentrate on systems of two differential equations.
The Lotka-Volterra Model

System	**Initial Conditions**
$F'(t)=aF-bF^2-cFS$	$F(0)=F_0$
$S'(t)=-kS+dSF$	$S(0)=S_0$

F(t) represents the population of the fish at time t
S(t) represents the population of the sharks at time t
F_0 is the initial size of the fish population
S_0 is the initial size of the shark population
Understanding the Model
$F'(t)=aF-bF^2-cFS$
F'(t) the growth rate of the fish population, is influenced, according to the first differential equation, by three different terms.
It is positively influenced by the current fish population size, as shown by the term aF, where a is a constant, non-negative real number and aF is the birthrate of the fish.

- It is negatively influenced by the natural death rate of the fish, as shown by the term $-bF^2$, where b is a constant, non-negative real number and bF^2 is the natural death rate of the fish
- It is also negatively influenced by the death rate of the fish due to consumption by sharks as shown by the term -cFS, where c is a constant non-negative real number and cFS is the death rate of the fish due to consumption by sharks.
- $S'(t)=-kS+dSF$
- S'(t), the growth rate of the Shark population, is influenced, according to the second differential equation, by two different terms.
- It is negatively influenced by the current shark population size as shown by the term -kS, where k is a constant non-negative real number and S is the shark population.

It is positively influenced by the shark-fish interactions as shown by the term dSF, where d is a constant non-negative real number, S is the shark population and F is the fish population

Equilibrium Points

- Once the initial equations are understood, the next step is to find the equilibrium points.
- These equilibrium points represent points on the graph of the function which are significant.

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- These are shown by the following computations.
- Let Y= (dS/dt)=S(-k+dF)
- To compute the equilibrium points we solve (dF/dt)=0 and (dS/dt)=0
- (dF/dt)=0 when F=0 or a-bF-cS=0
- solution: F=(a-cS)/b
- dS/dt=0 when S=0 or -k+dF=0
- Solution:{F=(k/d)}
- Now we find all the combinations:
- One of our equilibrium points is (0,0).
- For F=(a-cS)/b: When S=0, then F=((a-c(0))/b)=(a/b)
- Thus, one of our equilibrium points is ((a/b),0).
- For F=((a-cS)/b) and F=(k/d): (k/d)=((a-cS)/b),
- Solution is: {S=((-kb+ad)/(dc))}
- Thus, one of our equilibrium points is ((k/d),((-kb+ad)/(dc))).
- Our equilibrium points are (0,0), ((a/b),0), and
- ((k/d),((-kb+ad)/(dc))).

The first case has the fish population in a shark-free world.
In the second case, the shark conditions population will die out REGARDLESS of the initial !
For the third case the sharks die out again.
For the fourth case both the fish and shark populations wax and wane in a cyclic pattern with the sharks lagging behind the fish.

Carbon Dating

The key to the carbon dating of paintings and other materials such as fossils and rocks lies in the phenomenon of radioactivity discovered at the turn of the century.

The physicist Rutherford and his colleagues showed that the atoms of certain radioactive elements are unstable and that within a given time period a fixed portion of the atoms spontaneously disintegrate to form atoms of a new element.

Because radioactivity is a property of the atom, Rutherford showed that the radioactivity of a substance is directly proportional to the number of atoms of the substance present.

Thus, if N(t) denotes the number of atoms present at time t, then , the number of atoms that disintegrate per unit time, is proportional to N; that is,

$$dN/dt = -\lambda N$$

The half-life of many substances have been determined and are well published. For example, half-life of carbon-14 is 5568 years, and the half-life of uranium 238 is 4.5 billion years.

Connecting to Biological Reactions

- Consider transcription, the rate is constant and is not dependent upon the concentration of any of the species involve

$$\phi(DNA) \xrightarrow{Transcription} mRNA$$

$$\frac{d[mRNA]}{dt} = k_{trasc}$$

consider degradation of either mRNA or protein…

$$mRNA \xrightarrow{Degradation} \phi$$

$$\frac{d[mRNA]}{dt} = -\gamma_{\deg}[mRNA]$$

➢ Now consider the phenomenon of translation

$$\text{mRNA} \xrightarrow{\text{tRNA\&Ribosomes}} \text{protein}$$

The rate of translation is dependent upon the amount of mRNA

$$\frac{d[protein]}{dt} = k[mRNA]$$

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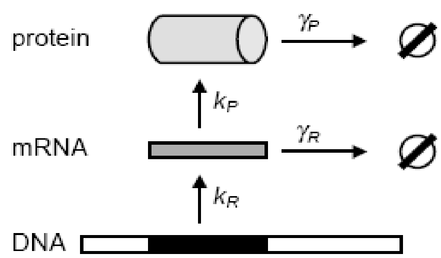
k is the rate constant to be determined experimentally

Combining Everything Together

- Now consider a protein synthesis reaction where the mRNA and protein are degraded:



$$\frac{d[protein]}{dt} = k_P[mRNA] - \gamma_P[protein]$$

$$\frac{d[mRNA]}{dt} = k_R - \gamma_R[mRNA]$$

First term is the transcription rate and the second term is the degradation rate.

Differential Equations in Forensic Science

For calculus in forensic biology, DNA sequences use power series as a powerful tool to compare DNA from crime scenes, for example.

To put it in simpler terms, power series are to functions what DNA molecules are to people.

Specifically for pathologists, calculus is needed to estimate the time of death for victims. Overall, calculus has many applications to many of the subfields and forensics and is often a useful tool in crime scene investigation.

DNA sequences are just nucleotide bases compared to a list of numbers forming a Taylor series.

The Taylor series serves as a "parent" function to the new function, which can be a derivative, antiderivative, similar series of DNA units can be related to offspring, parents, and other family members.

By determining the Taylor series of a DNA sequence, it can be compared to a standard DNA sequence for the database CODIS.

What forensic biologists try to figure out is what the power series of someone's DNA is, finding the function f(x), for which it is the Taylor series.

The similarities in the "Taylor series of related members can be perfectly calculated.

Draining a Tank

A tank in the form of a right-circular cylinder standing on end is leaking water through a circular hole in its bottom. Find the height h of water in the tank at any time t if the initial height of the water is H.

$$\frac{dh}{dt} = -\frac{B}{A}\sqrt{2gh}$$

where A is the cross section area of the cylinder and B is the cross sectional area of the orifice at the base of the container

$$\frac{dh}{\sqrt{h}} = -\frac{B}{A}\sqrt{2g}\quad dt$$

$$\frac{dh}{\sqrt{h}} = Cdt$$

$$C = -\frac{B}{A}\sqrt{2g}$$

By integrating this equation we get

$$2h^{\frac{1}{2}} = Ct + c'$$

For t=0 h=H and so
C=2Hexp1/2
h(t)=1/2(Ct + 2Hexp1/2)

Mathematics- Police Woman

The time of death of a murdered person can be determined with the help of modeling through differential equation.

- A police personnel discovers the body of a dead person presumably murdered and the problem is to estimate the time of death. The body is located in a room that is kept at a constant 70 degree F.
- For some time after the death, the body will radiate heat into the cooler room, causing the body's temperature to decrease assuming that the victim'semperature was normal 98.6F at the time of death.
- Forensic expert will try to estimate this time from body's current temperature and calculating how long it would have had to lose heat to reach this point.
- According to Newton's law of cooling, the body will radiate heat energy into the room at a rate proportional to the difference in temperature between the body and the room.

If T(t) is the body temperature at time t, then for some constant of proportionality k,
T'(t)=k[T(t)-70] This is a separable differential equation

$$\frac{1}{T-70}dT = kdt$$

Upon integrating both sides, one gets ln|T-70|=kt+c

Taking exponential, one gets

$|T-70| = e^{kt+C} = Ae^{kt}$

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where $A = e^C$.
Then $T-70= \pm Ae^{kt}= Be^{kt}$

Constants k and B can be determined provided the following information is available:

- Time of arrival of the police personnel,
- The temperature of the body just after his arrival,
- Temperature of the body after certain interval of time.

Let the officer arrive at 10.40 p.m. and the body temperature was 94.4 degrees. This means that if the officer considers 10:40 p.m. as t=0 then
$T(0)=94.4=70+B$ and so
$B=24.4$ giving
$T(t)=70 + 24.4\, e^{kt.}$

Let the officer makes another measurement of the temperature say after 90 minutes, that is, at 12.10 a.m. and temperature was 89 degrees.
This means that $T(90)=89=70+24.4\, e^{90k}$

Therefore, the time of death, according to this mathematical model, was
which is approximately –57.0.7 minutes.
The death occurred approximately 57.07 minutes before the first measurement at 10.40 p.m. , that is at 9.43 p.m. approximately

- **Drug Distribution (Concentration) in Human Body**

To combat the infection to human a body appropriate dose of medicine is essential. Because the amount of the drug in the human body decreases with time.

Medicine must be given in multiple doses. The rate at which the level y of the drug in a patient's blood decays can be modeled by the decay equation
$dy/dt=-ky$
where k is a constant to be experimentally determined for each drug.

If initially, that is, at t=0 a patient is given an initial dose y_p, then the drug level y at any time t is the solution of the above differential equations, that is,

$$y(t)=y_p\, e^{-kt}$$

Harvesting of Renewable Natural Resources

There are many renewable natural resources that humans desire to use. Examples are fishes in rivers and sea and trees from our forests. It is desirable that a policy be developed that will allow a maximal harvest of a renewable natural resource and yet not deplete that resource below a sustainable level.

A mathematical model providing some insights into the management of renewable resources.

Let P(t) denote the size of a population at time t, the model for exponential growth begins with the assumption that $dp/dt=kp$ for some $k>0$. In this model the relative or specific, growth rate defined by $dp/dt/p$ is assumed to be a constant.

In many cases it is not constant but a function of P, let $dp/dt/p = f(P)$

Suppose an environment is capable of sustaining no more than a fixed number K of individuals in its population. The quantity is called the carrying capacity of the environment.

Thus Differential equations and mathematical modeling can be used to study a wide range of social issues. Among the topics that have a natural fit with the mathematics in a course on ordinary differential equations are all aspects of population problems: growth of population, over-population, carrying capacity of an ecosystem, the effect of harvesting, such as hunting or fishing, on a population and how over-harvesting can lead to species extinction, interactions between multiple species populations, such as predator-prey, cooperative and competitive species.

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